Circulated Transport Phenomena driven by the Interaction of Arterial and Venous Vessel Systems simulated by Fractal Properties

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Abstract

Blood vessel system has attracted attention as a typical biological complex adaptive system. In this research, we investigate the properties of blood flow in fractal structure of retinal blood vessel systems modeled by diffusion-limited aggregation (DLA). It is found that there is a difference in the properties of blood flows between arterial and venous vessel systems, which originates from the fractal properties of their structures. This fact suggests that the difference in fractal structures between arterial and venous vessel systems should have something essential to do with the circulation of blood flow

1. Introduction

Blood vessel system is one of typical complex adaptive systems of a living body[1]. Fractal dimensions of structures in blood vessel systems of higher animals have been observed[2)-8)]. In recent years, the fact that the fractal dimension of the blood vessel systems varies with scales is reported in the literatures[7),8)]. It is expected that the fractal properties of blood vessel system have an effect on a function in the system. However, there are little researches on the correlation between the structural and the functional characteristics of the blood vessel systems.

In previous research, we are evaluated the fractal dimension as structural properties of generative

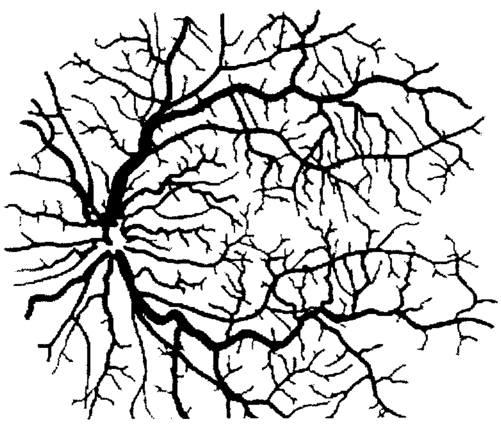

Fig. 1. A picture of retinal blood vessel system.

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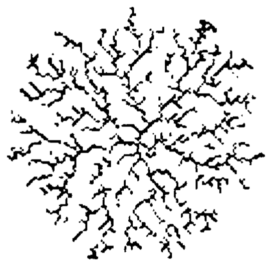

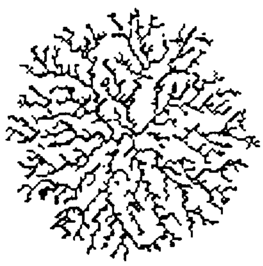

(a) (b)

Fig. 2. Retinal blood vessel patterns modeled by DLA.
(a) Pattern generated with *ds* = 8 corresponding to arterial blood vessel with fractal dimension 1.61.
(b) Pattern generated with *ds* = 6 corresponding to venous blood vessel with fractal dimension 1.65.

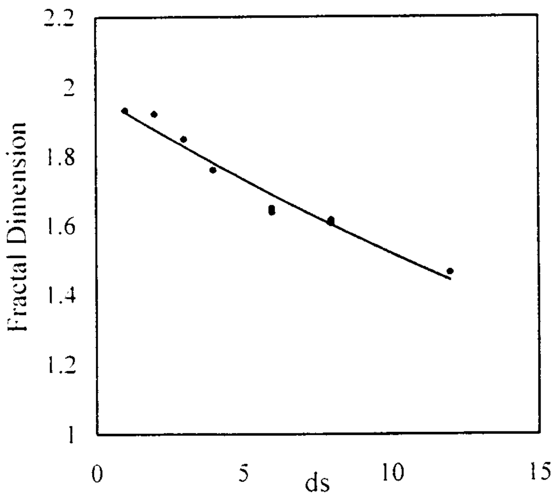

Fig. 3. Relationship of fractal dimension to a distance *ds* between particles composing DLA clusters.

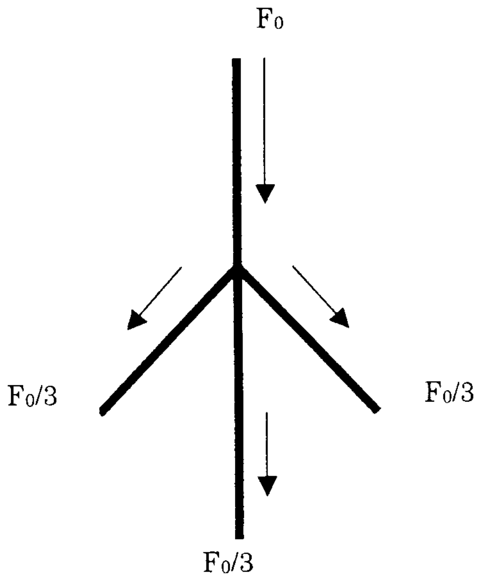

Fig. 4. Branching procedure of determination of flow in blood vessel system. Flow in a parent branch is divided into flows in daughter branches at equal weight.

cell growth simulated with hormonal proliferation algorithms[9]. In above investigation, we are indicated on fractal characterization of generative proliferation organisms. But, we are not enough understood at the function of structural fractal characterization in living organisms.

In this paper, in order to elucidate the effect of fractal structures of blood vessel systems on their function, we will investigate the properties of blood flow in the fractal structures of blood vessel systems modeled by diffusion-limited aggregation (DLA).

2. Materials and methods

We consider retinal blood vessels as a blood vessel systems having fractal structure. Fig. 1 shows an image of retinal blood vessel[2]. Fractal dimensions of arterial and venous blood vessels for the image are 1.61 and 1.65, respectively. Retinal arterial and venous patterns have a structure and fractal dimension similar to those of two-dimensional DLA[10]. In order to treat blood vessel patterns with various fractal dimensions, we model the fractal patterns of retinal blood vessels by DLA with various parameters controlling fractal dimensions.

DLA phenomena produce similar pattern whose non-equilibrium grows is governed by Laplace's equation and appropriate boundary condition. The algorithm of DLA is described as following procedures:

1. Put a seed particle on the origin (x_0, y_0) of the two-dimensional lattice.
2. Put another particle on a site far from the origin and make the particle do the random walk.
3. If the particle comes to the site (x, y) such that $/x\text{-}x_0/ = /y\text{-}y_0/ = ds$, put the particle on (x, y) and connect the two particles located at (x, y) and (x_0, y_0) with straight line. Here ds is a distance between particles composing DLA clusters.
4. To do repeat the procedure 2.
5. If the particle comes to the site (x, y) such that $/x\text{-}x_c/ = /y\text{-}y_c/ = ds$ where (x_c, y_c) is the site occupied a particle composing cluster, put the particle on (x, y) and connect the two particles located at (x, y) and (x_c, y_c) with straight line.
6. Repeat the procedures 4 and 5.

The pattern generated according to the algorithm shows branching structure similar to that of retinal blood vessel systems (Fig. 2). Note that the distance of particles ds controls fractal dimensions of structures. Fig. 3 shows the relationship between the fractal dimension and ds, which tells us that the fractal dimension decreases monotonously as ds increases. When $ds = 6$, DLA pattern has structure with fractal dimension similar to venous blood vessel, 1.65 (Fig. 2 (b)). On the other hand, DLA pattern generated with $ds = 8$ has structure with fractal dimension similar to arterial blood vessel, 1.61 (Fig. 2 (a)).

Blood flows in retinal blood vessel modeled by DLA are assumed to be determined by the *branching procedure*: flow in a parent branch (main branch) is divided into flows in daughter branches (sub branches) at equal weight (Fig. 4). We calculate blood flows in retinal blood vessel patterns modeled

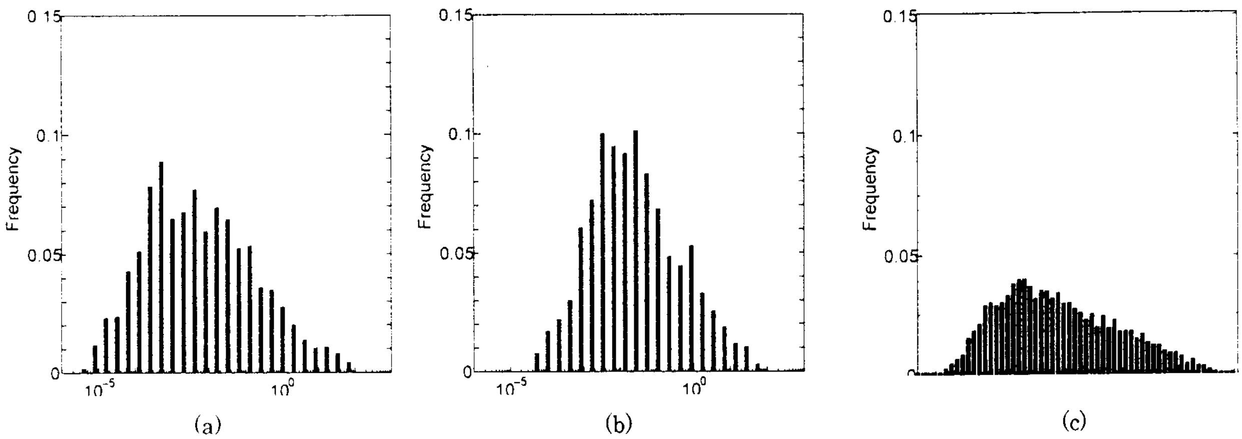

Fig. 5. Distribution of flow in blood vessel pattern modeled by DLA. These appeared flows are normalized to mean flow. The x axis is a logarithmic scale. (a) Arterial blood vessel pattern, (b) Venous blood vessel pattern, (c) Blood vessel pattern with fractal dimension 1.85.

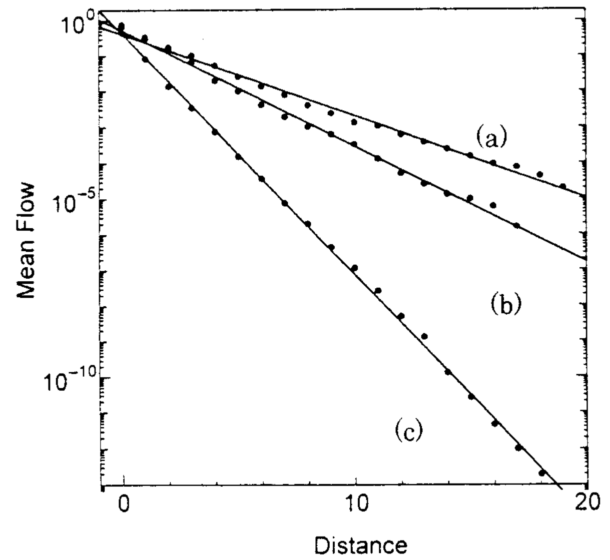

Fig. 6. The magnitude of mean flow as a function of the distance from the center of pattern. (a) Arterial pattern (b) Venous pattern (c) Pattern with fractal dimension 1.85.

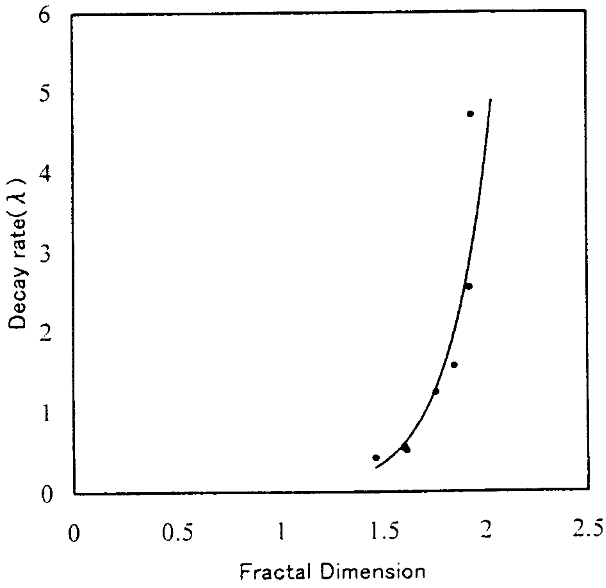

Fig.7. Relationship of decay rate to fractal dimension

by DLA with various fractal dimensions according to the branching procedure. In the simulation, the magnitude of flow at the center of pattern is set to 1.

3. Results

Fig. 5 shows the histogram of blood flows of blood vessel patterns modeled by DLA. Flows are normalized to mean flow. Note that x axis is a logarithmic scale. Fig. 5 (a) and (b) correspond to arterial and venous vessel patterns shown in Fig. 2 (a) and (b), respectively. The histogram of blood flows of blood vessel pattern with fractal dimension 1.85 is also shown in Fig. 5 (c). Flows of arterial system are distributed in 7-digit area and those of venous system in 8-digit. Compared with arterial and venous systems, flows of blood vessel patterns with fractal dimension 1.85 are distributed in wider range, 15-digit.

We observed that the magnitude of flow depends on the distance from the center of pattern. Fig. 6 shows the magnitude of mean flow as a function of the distance from the center of pattern. We found that the flows of blood vessel patterns decrease exponentially with increasing distance (flow$\sim e^{-\lambda r}$). The decay rate of the flows λ, obtained by the slope of the straight line on the semi log graph, for venous system is larger than that for arterial system. Furthermore, the decay rate of the flows in the pattern with fractal dimension 1.85 is larger than those of the other two systems.

Fig. 7 shows the relationship between fractal dimension and the decay rate of the flows. It is found that the decay rate of the flow increases monotonously with increasing fractal dimension.

4. Summary and Discussion

We evaluated blood flows in retinal blood vessel patterns modeled by DLA with various fractal dimensions including arterial and venous vessel systems with fractal dimensions 1.61 and 1.65, respectively. We found that the range of flow distribution for vessel patterns spreads with increasing fractal dimension. It was appeared that the magnitude of flow depends on the distance from the center of pattern, decreasing exponentially with increasing distance. The decay rate of flow for vessel patterns increase monotonously with increasing fractal dimension, which indicates that the decay rate of flow for venous system with fractal dimension 1.65 is larger than that for arterial system with fractal dimension 1.61.

These results indicate that the difference of fractal dimensions of blood vessel patterns influences their properties of flow. On the other hand, the difference in properties of spatial distribution of blood flow in vessel systems may amplify the fluctuation in circulation of blood. Therefore, it is suggested that the fractal structure of blood vessel system should have something essential to do with functions of blood vessel systems such as circulation driven by the difference in pressures between arterial and venous blood vessel systems.

From this research, we can derive the argument that difference of flow ratio in arterial and venous blood vessel systems is able to create two flows direction. Based on our simulation and analysis, we postulate the following hypothesis. The reason why introduce these results are that it give us reasonable theoretical background against circulation systems at a living blood vessel. Namely, it is that two difference flows are derive on circulated transport phenomena with arterial venous blood vessel systems. A further important point is that the issue of differences of fractal dimension may not be irrelevant to the issue of so-called adaptivity of blood flows under action of organization. A further more important point may be that the rules in blood circulation are adaptive based on variety of fractal structure at blood vessel system. that is to say, the proximity of arterial and venous blood vessels with fractal properties provides the complex spatial relationship that lead to diffusive interaction between paired arterial and venous blood vessel.

Consequently, it can be expected that the specific fractal structures observed in higher animal's blood vessel systems have effect on the control of the spatial distribution of blood flows adapted to the organization distribution.

In this paper, we do not yet considered a pulsatile effect (pumping effect of heart) of blood flows under these fractal properties. This effect is very interest for the function derived with fractal properties in blood vessel systems. Therefore, the following problem is to investigate the characterizations of blood flows in blood vessel systems taking account of its pulsatile effect. As further works, this investigation is in progress.

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